Shariah-Constrained Portfolio Construction

Measuring and Mitigating Structural Asset Concentration Risk

Abstract

Islamic finance has matured into a globally significant asset class, yet its portfolio construction framework remains fundamentally constrained by Shariah compliance requirements. These constraints comprising both qualitative sector exclusions and quantitative financial thresholds introduce non-random distortions into the investable universe. The resulting portfolios exhibit structural concentration effects that are not adequately captured by conventional portfolio theory.

This paper introduces a formal framework for identifying, measuring, and mitigating such concentration. It defines the Shariah Concentration Index (SCI) as a multi-dimensional metric capturing concentration across sectoral, asset-class, and balance-sheet dimensions. Unlike traditional concentration measures, SCI explicitly accounts for constraint-induced bias.

The paper further extends classical mean-variance optimization into a multi-objective formulation incorporating concentration as a first-class variable. Finally, it outlines the HalalCheck system architecture, which operationalizes these models in real-time institutional environments. The central contribution of the paper is the repositioning of concentration as a structural property of compliance rather than a by-product of allocation decisions, enabling a shift toward optimization-driven Islamic portfolio construction.

1. Introduction

The rapid growth of Islamic finance has led to increased institutional participation across equities, sukuk, and selected alternatives. Central to this ecosystem is adherence to Shariah principles, which govern both permissible economic activities and acceptable financial structures. In practice, these principles are implemented through a screening process that removes non-permissible sectors and excludes companies that fail financial ratio thresholds.

Shariah screening operates along two axes. First, qualitative constraints exclude industries such as conventional banking and insurance, alcohol, gambling, tobacco, and certain entertainment activities. Second, quantitative constraints filter companies based on leverage, interest-derived income, and other financial ratios. While these controls are essential for compliance, they also shrink the investable universe in a highly non-random way.

That distortion creates a critical portfolio-construction problem. Capital that would ordinarily spread across the full market is redirected into a narrower subset of sectors, asset classes, and financial profiles. This means a portfolio can look diversified when viewed through conventional line-item counts or even sector allocations yet still carry hidden structural concentration. The objective of this paper is to define that problem formally and present an institutional framework for measuring, monitoring, and mitigating it.

2. Literature Review

The literature on Islamic finance has historically focused on three broad areas: screening methodologies, relative performance, and constrained optimization. Methodology papers from AAOIFI, FTSE Russell, MSCI, and S&P Dow Jones establish the core principles and thresholds used to classify assets as Shariah-compliant. These frameworks provide the compliance rules that make Islamic investing possible, but they are not designed as concentration models. Their primary objective is eligibility, not diversification efficiency.

A second stream of work compares Islamic and conventional indices. Studies such as Hassan and Girard (2011) and Białkowski et al. (2013) document performance differences, sector bias, and leverage-related effects in Islamic portfolios. These studies show that Islamic strategies can perform competitively, but they also reveal recurring structural tilts toward lower leverage and away from financials. What they stop short of doing is treating concentration as a formal, multi-dimensional outcome of the compliance process itself.

A third-stream addresses portfolio optimization under Islamic constraints. Derigs and Marzban (2008) explored optimization in an Islamic setting and demonstrated that constrained portfolio construction is feasible. However, the optimization focus remained centered on return and risk under a restricted universe. The missing piece is a framework that formally measures concentration across multiple dimensions and embeds that measurement into optimization logic. This paper addresses that gap by introducing a Shariah-specific concentration metric and integrating it directly into the portfolio decision process.

3. Problem Definition

In conventional portfolio theory, concentration is usually interpreted as the result of manager choice. A portfolio is concentrated because capital has been intentionally or unintentionally allocated into too few names, sectors, or risk factors. In Shariah-compliant investing, however, concentration is not merely a choice variable. It is also a constraint-induced phenomenon.

Three distinct forms of structural concentration emerge. The first is sector concentration. Because financial services and other prohibited industries are excluded, the remaining compliant universe absorbs a greater share of portfolio capital. This naturally overweight's sectors such as technology, healthcare, and certain industrials. The second is asset-class concentration. The universe of compliant instruments outside equities and sukuk is still comparatively underdeveloped, which can leave portfolios structurally skewed toward a limited set of asset classes. The third is balance-sheet concentration. Financial filters exclude highly leveraged firms and businesses with excessive non-permissible income, leaving a narrower set of companies with similar financial characteristics.

These concentration effects interact. A portfolio may appear diversified by number of holdings while still being concentrated in underlying economic exposure, financing profile, or asset-type reliance. Traditional concentration metrics only partially detect this. A more suitable framework must measure concentration across the very dimensions that Shariah screening distorts. That requirement motivates the Shariah Concentration Index.

4. Concentration Measurement Framework: Shariah Concentration Index (SCI)

4.1 Conceptual Basis

Traditional portfolio concentration measures, including the Herfindahl-Hirschman Index (HHI), are useful for measuring observed concentration within a single dimension, such as issuer weights or sector weights. Their limitation in a Shariah context is that they do not explicitly capture the interaction between compliance screening and structural bias. A compliant portfolio may not look concentrated on one dimension alone, yet may still be concentrated when sector, asset, and financial-profile clustering are considered together.

The Shariah Concentration Index (SCI) addresses this by constructing a composite concentration measure across the three dimensions most directly affected by compliance screening: sector, asset class, and balance-sheet structure.

4.2 Formal Definition

$$SCI = \alpha C\_sector + \beta C\_asset + \gamma C\_balance$$

Where C_sector is sector concentration, C_asset is asset-class concentration, and C_balance is balance-sheet concentration. The coefficients α, β, and γ are weighting parameters that sum to 1. They can be set equally for a neutral baseline or adjusted to reflect mandate-specific priorities.

4.3 Component 1: Sector Concentration

$$C\_sector = \Sigma(w\_i^2)$$

Here, w_i represents the portfolio weight allocated to sector i. The metric increases as capital becomes concentrated into fewer sectors. In a Shariah context, the exclusion of conventional financials and other prohibited activities means the investable universe is already tilted. This causes remaining sectors to absorb disproportionate weight, even before active portfolio choices are made.

4.4 Component 2: Asset-Class Concentration

$$C\_asset = \Sigma(a\_j^2)$$

Here, a_j represents the allocation to asset class j, such as equities, sukuk, or alternatives. This component captures the reality that a portfolio may be diversified inside an asset class while still being concentrated at the portfolio level because too much capital is tied to one broad category. For many current Islamic portfolios, this dimension is especially important because the opportunity set outside equities and sukuk remains unevenly developed.

4.5 Component 3: Balance-Sheet Concentration

$$C\_balance = \Sigma(b\_k^2)$$

Here, b_k represents the proportion of portfolio exposure within financial-profile cluster k. These clusters can be defined using debt ratios, liquidity levels, interest-income exposure, or related balance-sheet features. The purpose of this component is to capture hidden homogeneity. Two firms in different sectors may still look very similar once their financing structure is considered. Shariah screening often favors low-leverage firms, which can create an under-recognized clustering effect across the compliant universe.

4.6 Weighting Strategy

A baseline specification uses equal weights: $\alpha = \beta = \gamma = 1/3$. This is appropriate where the objective is broad institutional comparability. However, the weighting scheme can be adapted. An equity-heavy mandate may place greater weight on sector concentration, a multi-asset mandate may prioritize asset-class concentration, and a stricter screening policy may increase the importance of balance-sheet concentration. The weighting framework therefore allows SCI to serve both as a standardized institutional metric and as a configurable mandate-specific control.

4.7 Interpretation Bands

SCI score	Interpretation	Risk implication
0.00 – 0.20	Highly diversified	Low structural concentration
0.20 – 0.40	Moderately diversified	Manageable concentration
0.40 – 0.60	Concentrated	Elevated structural concentration
0.60 – 1.00	Highly concentrated	High structural concentration risk

4.8 Worked Example

Consider a portfolio with 60% allocated to technology, 20% to healthcare, and 20% to industrials. Sector concentration is therefore $0.6^2 + 0.2^2 + 0.2^2 = 0.44$. Assume the same portfolio holds 80% equities and 20% sukuk, producing asset-class concentration of $0.8^2 + 0.2^2 = 0.68$. If 90% of holdings fall into a low-leverage balance-sheet cluster and 10% into a moderate-leverage cluster, then balance-sheet concentration is $0.9^2 + 0.1^2 = 0.82$.

$$SCI = (0.44 + 0.68 + 0.82) / 3 = 0.647$$

An SCI of 0.647 places the portfolio in the highly concentrated category. The key insight is that a portfolio that may appear diversified by number of holdings can still demonstrate significant structural concentration once all three dimensions are measured together.

4.9 Operational Use of SCI

SCI is not merely a descriptive statistic. It can be used in three ways. First, as a monitoring metric, it provides an institutional dashboard signal for concentration drift over time. Second, as a hard constraint, it can cap the allowable concentration level in portfolio construction. Third, as an optimization penalty, it can be incorporated directly into the objective function to penalize structurally concentrated solutions. This is what turns SCI from a diagnostic measure into an actionable portfolio-construction tool.

5. Shariah-Constrained Portfolio Optimization Engine

5.1 Baseline Formulation

Classical mean-variance optimization seeks to maximize expected return for a given level of risk. In compact form, the problem may be written as maximizing expected return minus a risk penalty. That framework assumes a fully investable universe and does not explicitly account for structural concentration. Those assumptions do not hold in Shariah-compliant investing.

$$Maximize: E[R_p] - \lambda\sigma_p^2 - \mu \cdot SCI(w)$$

In this formulation, $E[R_p]$ denotes expected portfolio return, σ_p^2 denotes portfolio variance, $SCI(w)$ is the Shariah Concentration Index as a function of portfolio weights, λ is the risk-aversion coefficient, and μ is the concentration-penalty coefficient. The concentration term converts optimization from a two-dimensional trade-off into a three-dimensional one balancing return, volatility, and structural concentration.

5.2 Constraint Set

- Budget constraint: total portfolio weights sum to 1.
- Long-only constraint where required: portfolio weights remain non-negative.
- Shariah eligibility constraint: non-compliant assets receive zero weight.
- Sector caps to prevent excessive concentration in any one industry.
- Asset-class caps to control overreliance on equities, sukuk, or other categories.
- SCI threshold to restrict maximum allowable structural concentration.

5.3 Why SCI Changes the Optimization Problem

Because SCI is built from concentration components that depend on portfolio weights, it introduces an additional structural dimension into the feasible set. The resulting efficient frontier is no longer a simple risk-return curve. It becomes a risk-return-concentration surface. This matters operationally because it allows portfolio managers to choose solutions that trade a modest amount of expected return for materially better diversification quality, rather than accepting structural concentration as an unavoidable by-product of compliance.

5.4 Dynamic Optimization

Shariah-compliant portfolios cannot be treated as static optimization exercises. Prices change, financial ratios move, corporate actions alter compliance status, and sector classifications can shift. The optimization problem therefore needs to be solved repeatedly across time, with the compliant universe and concentration profile recalculated as inputs change. In practical terms, this means optimal weights are time-dependent and portfolio construction becomes a continuous monitoring and rebalancing exercise rather than a periodic screen-and-hold process.

5.5 Implementation Approaches

Several implementation approaches are viable. Quadratic programming is suitable where the optimization can be expressed in a tractable convex form. Multi-objective optimization frameworks are useful when explicit Pareto trade-offs between return, risk, and concentration need to be explored. Heuristic methods such as genetic algorithms may be appropriate when the problem becomes highly non-linear or involves a large, complex universe. The choice of method depends on institution size, frequency of rebalancing, and infrastructure maturity, but the conceptual point remains the same: concentration must be explicitly optimized, not passively tolerated.

Parameter	Meaning	Portfolio effect when increased
λ	Risk-aversion coefficient	Pushes portfolio toward lower variance
μ	Concentration-penalty coefficient	Pushes portfolio toward lower SCI
δ	Maximum SCI threshold	Hard-caps structural concentration

6. HalalCheck System Architecture

6.1 Architectural Objective

HalalCheck is designed to operationalize the measurement and optimization framework in an institutional setting. The architecture must therefore support real-time or near-real-time analytics, auditable compliance logic, scalable data ingestion, and explainable outputs suitable for portfolio managers, risk teams, boards, and Shariah supervisors.

6.2 Layered Architecture

Layer	Purpose	Core outputs
Data layer	Ingest prices, fundamentals, classifications, and rule inputs	Normalized market and compliance data
Screening engine	Apply qualitative and quantitative Shariah filters	Eligible universe
SCI engine	Calculate concentration across sector, asset, and balance dimensions	SCI score and sub-scores
Optimization engine	Construct or rebalance portfolios under constraints	Target weights and trade recommendations
API layer	Expose scoring and optimization services to enterprise systems	Structured machine-readable outputs
Dashboard layer	Provide visual analytics and oversight	Monitoring, reporting, and governance views

6.3 Data Layer

The data layer must ingest both financial and compliance-relevant data. This includes market prices, issuer fundamentals, sector classifications, and rule datasets required for screening. The critical design principle is that data should not merely support eligibility decisions; it must also support modelling of concentration and optimization. That means the system must preserve enough granularity to identify balance-sheet clusters, monitor sector migration, and compute concentration drift over time.

6.4 Screening Engine

The screening engine operationalizes the Shariah rulebook. It applies qualitative exclusions for prohibited sectors and quantitative thresholds for leverage, non-permissible income, and related financial measures. The output is a compliant investable universe at time t. The importance of this layer is not only that it defines eligibility, but that it also generates the constrained universe on which all subsequent concentration and optimization calculations depend.

6.5 SCI Engine

The SCI engine computes the sector, asset-class, and balance-sheet concentration components and combines them into a portfolio-level score. It must be able to calculate both absolute SCI and the contribution of each component so that managers can see whether concentration risk is being driven primarily by sector skew, overreliance on one asset class, or excessive financial-profile homogeneity. This decomposition is essential for actionable remediation.

For institutional use, the SCI engine should support continuous recalculation, scenario analysis, and historical tracking. This allows concentration to be monitored as a time series rather than as a one-off diagnostic.

6.6 Optimization Engine

The optimization engine consumes the compliant universe, the SCI sub-components, and portfolio constraints to generate target allocations or rebalance recommendations. It is the decision layer of the architecture. Rather than simply reporting concentration, it generates a portfolio design response to it. In mature deployments, this engine can support both full optimization and constrained what-if simulations so that managers can test alternative mandates, sector caps, or SCI thresholds before execution.

6.7 API and Dashboard Layers

The API layer exposes portfolio scoring, compliance checking, and optimization outputs to surrounding enterprise systems, including portfolio management tools, order workflows, and reporting environments. The dashboard layer translates those outputs into governance-ready views. Required dashboard elements include SCI trend lines, sector and asset breakdowns, balance-profile clustering, compliance exceptions, and before-versus-after optimization comparisons. Institutional credibility depends heavily on explainability; users must be able to see not just the score, but why the score moved and what actions are being recommended.

7. Use Cases and Case Studies

7.1 Use Cases

- Asset managers seeking to improve diversification quality inside a Shariah-compliant mandate.
- Islamic banks and treasury teams monitoring concentration across sukuk and related compliant exposures.
- Shariah boards requiring auditable evidence that a portfolio is not only compliant but prudently diversified.
- Investor and regulator-facing teams needing a defensible narrative for how concentration is measured and controlled.

7.2 Case Study 1: Equity-Dominated Portfolio

Assume a portfolio initially allocated 75% to equities, with a heavy bias toward technology and other growth sectors, and with most holdings passing Shariah thresholds through low leverage. On conventional views, the portfolio appears acceptably diversified because it holds multiple issuers. However, once measured through SCI, it scores 0.66, reflecting high structural concentration. The dominant drivers are sector overweight and balance-sheet clustering.

Applying the optimization framework with sector caps, a stronger concentration penalty, and a broader compliant opportunity set reduces equity dominance and improves diversification across sectors and balance profiles. The resulting SCI falls to 0.39. The significance of the case is that diversification improves not simply by increasing the number of holdings, but by changing the structure of exposures that compliance had compressed.

7.3 Case Study 2: Sukuk-Dominated Portfolio

Consider a second portfolio with 85% allocated to sukuk and minimal exposure to compliant equities or alternatives. Such a portfolio may suit a defensive mandate but can still be structurally concentrated from an asset-class perspective. SCI identifies this by producing a score of 0.58, with asset-class concentration as the dominant component.

Introducing a controlled allocation to compliant equities and selected alternatives, while preserving mandate integrity, lowers SCI to 0.41. The lesson is that concentration can arise in either direction: not only from equity overdependence, but also from an over-concentration in sukuk. SCI provides a consistent framework for evaluating both situations.

Case study	Initial SCI	Post-optimization SCI	Primary improvement driver
Equity-dominated portfolio	0.66	0.39	Sector and balance diversification
Sukuk-dominated portfolio	0.58	0.41	Asset-class diversification

8. Conclusion

Shariah compliance creates a portfolio-construction problem that is deeper than simple screening. By excluding sectors and filtering firms through financial thresholds, it introduces structural concentration across the very dimensions that matter most for diversification. This concentration is not incidental; it is embedded in the compliant opportunity set itself.

The Shariah Concentration Index provides a practical and formal way to measure that effect. The optimization framework makes concentration actionable by incorporating it directly into portfolio design. The HalalCheck architecture then operationalizes the full methodology in a production environment. Taken together, these elements support a strategic shift in Islamic finance: away from static pass/fail compliance as the end state, and toward optimization-driven, intelligence-enabled portfolio construction as the new standard.

Appendices

A1. SCI Formula Summary

$$SCI = \alpha C\_sector + \beta C\_asset + \gamma C\_balance$$

$$C\_sector = \Sigma(w\_i^2)$$

$$C\_asset = \Sigma(a\_j^2)$$

$$C\_balance = \Sigma(b\_k^2)$$

These formulae provide the baseline specification for SCI. The weighting parameters can be standardized across the institution or adjusted to reflect mandate-specific priorities.

A2. Optimization Formulation

$$Maximize: E[R\_p] - \lambda\sigma\_p^2 - \mu \cdot SCI(w)$$

This objective function captures the core three-way trade-off between expected return, volatility, and structural concentration. It can be used in hard-constraint or penalty-based form depending on implementation choice.

A3. Parameter Definitions

Parameter	Definition
α, β, γ	SCI component weights for sector, asset class, and balance-sheet concentration
λ	Risk-aversion coefficient in optimization
μ	Concentration-penalty coefficient in optimization
δ	Maximum SCI threshold where a hard cap is used

A4. Example Calculation Notes

The worked example in Section 4 is intentionally simple so that SCI remains transparent and explainable to non-quantitative stakeholders. In production use, the same logic can be extended to larger universes, richer balance-sheet clustering schemes, and scenario-based concentration decomposition.

A5. Implementation Notes

Institutional implementation can be achieved through quadratic programming, multi-objective optimization, or heuristic methods depending on complexity and scale. The methodological priority is not the specific solver but the explicit inclusion of concentration as a measurable portfolio variable and a controllable optimization input.

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